

A Real CKM Matrix and Physics Beyond the Standard Model

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Abstract

We study the possible existence of a real Cabibbo-Kobayashi-Maskawa (CKM) matrix, with CP violation originating from physics beyond the standard model (SM). We show that present experimental data allow for a real CKM matrix provided that new physics also contributes to Δm_{B_d} by at least 20% of the SM contribution (for $\rho > 0$), besides generating CP violation in the kaon sector. The naturalness of a real CKM matrix is studied within the framework of general multi-Higgs-doublet models with spontaneous CP violation. As an example, we discuss a specific two-Higgs-doublet model and its implications for CP asymmetries in non-leptonic neutral B -meson decays.

1 Introduction

At present all information on CP violation is consistent with the standard model (SM) of electroweak interactions and its Cabibbo-Kobayashi-Maskawa (CKM) mechanism of CP breaking [1].

In this Letter, we address the question of whether or not current experimental data, in particular the new limit for the B_s^0 oscillation frequency, exclude the possibility of having a real CKM matrix. We envisage a scenario where charged weak interactions are CP conserving, while all CP-violating phenomena stem from physics beyond the SM. Along with unitarity, the CKM matrix V_{CKM}

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is already constrained by experimental data; in particular strange particle and B -meson decays lead to the measurement of the moduli of the CKM matrix elements $|V_{us}|$, $|V_{cb}|$, and $|V_{ub}/V_{cb}|$. Since the new physics contribution to the above decays in the most plausible extensions of the standard theory cannot compete with the SM tree-level processes, the determination of these matrix elements is likely to hold, even in the presence of physics beyond the SM [2]. Further constraints on the CKM matrix arise from the strength of CP violation in the kaon sector measured by $|\epsilon_K|$ and from the observation of B_d^0 - \bar{B}_d^0 mixing, along with the improved experimental lower limit of Δm_{B_s} . Within the SM, the experimental results for Δm_{B_d} and Δm_{B_s} can be used to extract the values of $|V_{td}|$ and $|V_{ts}|$, while the measured value of $|\epsilon_K|$ directly constrains the size of the CKM phase.

What follows is a simple method of checking whether or not present experimental data already exclude the possibility of having a real CKM matrix. Let us assume that V_{CKM} is real¹ and that the observed CP violation in the kaon sector arises solely from physics beyond the SM. The experimental information on $|V_{us}|$, $|V_{cb}|$, and $|V_{ub}/V_{cb}|$, together with the unitarity of V_{CKM} , allows one to deduce the values for $|V_{td}|$ and $|V_{ts}|$. Moreover, a comparison of these values with those extracted from Δm_{B_d} and Δm_{B_s} measurements enables us to ascertain the viability of a real CKM matrix. One may be tempted to interpret any incompatibility between the two sets of $|V_{td}|$ and $|V_{ts}|$ values obtained above as an indication that the CKM matrix cannot be real. However, the situation is more involved for the following reason. Within the SM, Δm_{B_d} and Δm_{B_s} are generated only at one-loop level by the W -mediated box diagrams, and as a result, physics beyond the SM can significantly contribute to Δm_{B_d} and Δm_{B_s} . Indeed, in many extensions of the standard theory, the same new physics that gives rise to ϵ_K is likely to provide significant contributions to Δm_{B_d} and Δm_{B_s} .

In this paper, we first analyse the somewhat unnatural situation where new physics contributes only to ϵ_K and not to the mass difference in the B system. We show that in this case the currently available data already disfavour a real CKM matrix, thus confirming previous results [4,5]. We then investigate the more plausible situation of a real CKM matrix, with new physics generating ϵ_K and also contributing to Δm_{B_d} and Δm_{B_s} . In this case, as we shall see below, present experimental data are compatible with the scenario of a real CKM matrix.

Our paper is organized as follows. In Sec. 2, we present a general analysis of $\Delta B = 2$ transitions in the presence of new physics. Our results are model independent and apply to a wide class of models with a real CKM matrix. Section 3 deals with multi-Higgs-doublet models in which CP is softly broken.

¹ This case has also been discussed, for example, in Ref. [3].

Assuming that CP is a symmetry of the Lagrangian, spontaneously broken by the vacuum, we discuss the possibility of having a real CKM matrix in a natural way. In Sec. 4, we illustrate how the experimental constraints on ϵ_K , Δm_K , and Δm_{B_d} can be satisfied with a real CKM matrix in the context of a simple model with two Higgs doublets, and analyse the predictions for CP asymmetries in non-leptonic neutral B decays that will be measured in upcoming B experiments. Finally, we present our conclusions in Sec. 5.

2 General analysis

2.1 Basic formulae and notation

We first carry out a general analysis of the new physics contribution to $\Delta B = 2$ transitions (see, e.g., Refs. [6,7]). The off-diagonal element of the B_q mass matrix can be written as²

$$M_{12}(B_q) \equiv M_{12}^{\text{New}}(B_q) + M_{12}^{\text{SM}}(B_q) = \frac{\langle B_q | H_{\text{eff}} | \bar{B}_q \rangle}{2m_{B_q}}, \quad (1)$$

where $q = d$ or s , and the effective Hamiltonian has the form

$$H_{\text{eff}} = H_{\text{eff}}^{\text{SM}} + H_{\text{eff}}^{\text{New}}. \quad (2)$$

It is customary to parametrize the new physics contribution appearing in Eq. (1) through

$$r_q^2 \exp(2i\theta_q) \equiv \frac{M_{12}(B_q)}{M_{12}^{\text{SM}}(B_q)}, \quad (3)$$

so that

$$\Delta m_{B_q} = 2|M_{12}^{\text{SM}}|_q r_q^2. \quad (4)$$

Note that $r_q^2 < 1$ arises if the new physics amplitudes interfere destructively with those of the SM. The contribution of the SM to B_q^0 - \bar{B}_q^0 mixing is given by

$$M_{12}^{\text{SM}}(B_q) = \frac{G_F^2}{12\pi^2} \eta_B m_{B_q} m_W^2 f_{B_q}^2 B_{B_q} S_0(x_t) (V_{tq} V_{tb}^*)^2, \quad x_t = \bar{m}_t^2/m_W^2, \quad (5)$$

² For notational simplicity, we will omit the q subscript for H_{eff} .

where the various factors appearing in Eq. (5) may be found elsewhere [8,9]. In fact, we have

$$M_{12}^{\text{SM}}(B_d) = 3.1 \times 10^3 \text{ ps}^{-1} \left[\frac{f_{B_d} \sqrt{B_{B_d}}}{200 \text{ MeV}} \right]^2 \left[\frac{S_0(x_t)}{2.36} \right] \left[\frac{\eta_B}{0.55} \right] (V_{td} V_{tb}^*)^2, \quad (6)$$

and

$$M_{12}^{\text{SM}}(B_s) = 4.5 \times 10^3 \text{ ps}^{-1} \left[\frac{f_{B_s} \sqrt{B_{B_s}}}{240 \text{ MeV}} \right]^2 \left[\frac{S_0(x_t)}{2.36} \right] \left[\frac{\eta_B}{0.55} \right] (V_{ts} V_{tb}^*)^2. \quad (7)$$

Introducing the SU(3) breaking parameter

$$\xi_s \equiv \frac{f_{B_s} \sqrt{B_{B_s}}}{f_{B_d} \sqrt{B_{B_d}}}, \quad (8)$$

we find the ratio

$$\frac{\Delta m_{B_s}}{\Delta m_{B_d}} = \xi_s^2 \frac{m_{B_s}}{m_{B_d}} \left| \frac{V_{ts}}{V_{td}} \right|^2 \frac{r_s^2}{r_d^2}. \quad (9)$$

Finally, the CP-violating asymmetry between B^0 and \bar{B}^0 mesons decaying to CP eigenstates f_{CP} , such as $J/\psi K_S^0$ or $\pi^+ \pi^-$, in the presence of new physics is given by

$$a_{f_{\text{CP}}}(t) = \frac{\Gamma(B_d^0(t) \rightarrow f_{\text{CP}}) - \Gamma(\bar{B}_d^0(t) \rightarrow f_{\text{CP}})}{\Gamma(B_d^0(t) \rightarrow f_{\text{CP}}) + \Gamma(\bar{B}_d^0(t) \rightarrow f_{\text{CP}})} = -a_{f_{\text{CP}}} \sin(\Delta m_{B_d} t), \quad (10)$$

with

$$a_{J/\psi K_S^0} = \sin 2(\beta + \theta_d + \theta_K), \quad a_{\pi^+ \pi^-} = \sin 2(\alpha - \theta_d). \quad (11)$$

Here we have ignored penguin contributions as well as final-state interaction phases. In addition, we have assumed that the direct decay amplitudes are dominated by the SM tree diagrams. The phase θ_K in the above expression is caused by the new physics contribution (normalized to the SM amplitude) to $K^0 - \bar{K}^0$ mixing, i.e. the mixing parameter $(q/p)_K$ picks up another phase. However, given the experimental information on ϵ_K , we expect the phase θ_K to be of $\mathcal{O}(10^{-3})$. We will return to this point in Sec. 4.

2.2 New physics contribution and experimental data

As mentioned above, we will deal with a real CKM matrix. In this case, we have

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \lambda |\rho| , \quad (12)$$

and (neglecting higher order terms in λ)

$$V_{td} \simeq A\lambda^3 \left[1 - \rho \left(1 - \frac{1}{2}\lambda^2 \right) \right] , \quad V_{ts} \simeq -A\lambda^2 \left[1 - \frac{1}{2}\lambda^2(1 - 2\rho) \right] , \quad (13)$$

with the Wolfenstein parameters A , λ , and ρ [10]. In our numerical analysis, we use the following input parameters [6,9,11–13]:

$$\left| \frac{V_{ub}}{V_{cb}} \right| = \left| \frac{V_{ub}}{V_{cb}} \right|_{\text{T}} \pm 0.005, \quad |V_{cb}| = 0.0395 \pm 0.0017 , \quad (14)$$

$$A = 0.819 \pm 0.035, \quad \lambda = 0.2196 \pm 0.0023, \quad \overline{m}_t = 167 \pm 6 \text{ GeV} , \quad (15)$$

$$|\epsilon_K| = (2.280 \pm 0.019) \times 10^{-3} , \quad (16)$$

$$\Delta m_K = (0.5301 \pm 0.0014) \times 10^{-2} \text{ ps}^{-1} , \quad (17)$$

$$\Delta m_{B_d} = 0.471 \pm 0.016 \text{ ps}^{-1}, \quad \Delta m_{B_s} > 12.4 \text{ ps}^{-1} \text{ (95\% C.L.)} , \quad (18)$$

and consider the ranges

$$0.06 \leq \left| \frac{V_{ub}}{V_{cb}} \right|_{\text{T}} \leq 0.10, \quad 160 \text{ MeV} \leq f_{B_d} \sqrt{B_d} \leq 240 \text{ MeV} , \quad (19)$$

$$1.12 \leq \xi_s^2 \leq 1.48 . \quad (20)$$

By using the experimentally measured values for $|V_{ub}/V_{cb}|$, $|V_{cb}|$, and Δm_{B_d} (ignoring for the moment the constraint imposed by the B_s system), we can perform a fit with two parameters ρ and r_d^2 . The results of the fit are shown in Fig. 1, from which we infer the upper and lower limits

$$0.24 \lesssim \rho \lesssim 0.49, \quad 1.2 \lesssim r_d^2 \lesssim 6.5 , \quad (21a)$$

$$-0.49 \lesssim \rho \lesssim -0.24, \quad 0.3 \lesssim r_d^2 \lesssim 1.1 . \quad (21b)$$

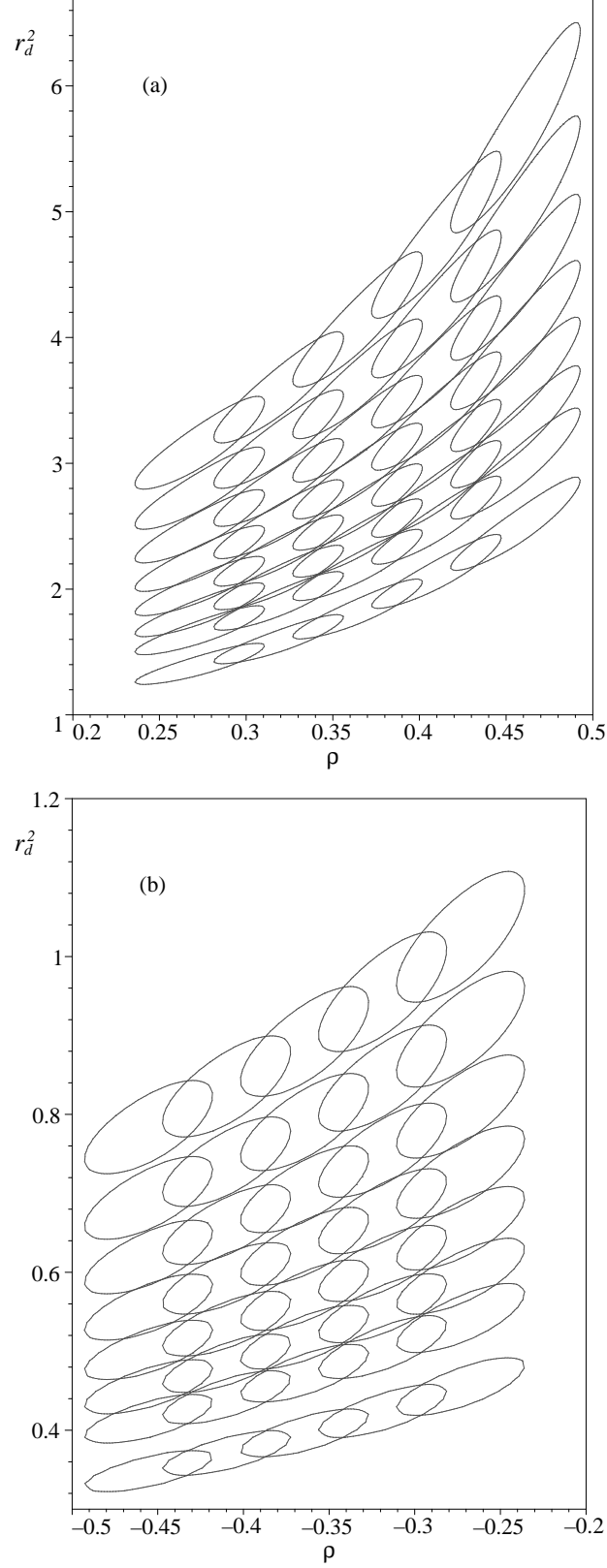


Fig. 1. The allowed region (at 95% C.L.) in the (ρ, r_d^2) plane for the Wolfenstein parameter $\rho > 0$ (a) and $\rho < 0$ (b), using the experimental values for $|V_{ub}/V_{cb}|$, $|V_{cb}|$, and Δm_{B_d} . Each contour corresponds to a variation of the theoretical input parameters, as described in the text [cf. Eq. (19)].

Thus, for positive ρ values, a new physics contribution to Δm_{B_d} of at least 20% of the SM contribution is required for consistency with current experimental data. Including the Δm_{B_s} constraint, our analysis gives the lower bound as

$$\rho > 1 - \sqrt{C_s \frac{r_s^2}{r_d^2}}, \quad C_s = \frac{1}{\lambda^2} \frac{m_{B_s}}{m_{B_d}} \frac{\Delta m_{B_d}}{\Delta m_{B_s}} \xi_s^2 \leq 0.8 \xi_s^2. \quad (22)$$

Two remarks are in order here. First, it is important to note that without new physics contributions to Δm_{B_d} and Δm_{B_s} , i.e. $r_d^2 = r_s^2 = 1$, we obtain from Eq. (22) $\rho > -0.08$. By using the bound of Eq. (21), one immediately sees that a real CKM matrix is disfavoured by present experimental data, as also reported in Refs. [4,5]. Second, for $r_s^2 = 1$ but $r_d^2 \neq 1$, it follows from Eqs. (21) and (22) that either negative or positive values for ρ are allowed by current experimental data. That is, a real CKM matrix is still consistent with the above measurements provided new physics contributes to both the K and B_d^0 system.

3 A real CKM matrix

3.1 Multi-Higgs-doublet models

This section is concerned with the question of how to construct models that naturally lead to a real CKM matrix, within the framework of multi-Higgs-doublet models (MHDMs).³

The most general Yukawa couplings in the MHDM are given by the following Lagrangian (in the weak eigenbasis)

$$\mathcal{L}_Y = - \sum_{j=1}^M \left(\bar{Q}_L \Gamma_j \phi_j n_R + \bar{Q}_L \tilde{\Gamma}_j \tilde{\phi}_j p_R \right) + \text{H.c.} , \quad (23)$$

with the left-handed quark doublet $Q_L = (p_L, n_L)^T$ and the right-handed quark singlets p_R and n_R , where p and n denote the up- and down-type quarks respectively. Γ_j and $\tilde{\Gamma}_j$ are matrices in flavour space, and the Higgs doublets are given by $\phi_j = (\phi_j^+, \phi_j^0)^T$, $\tilde{\phi}_j = i\sigma_2 \phi_j^*$, with $j = 1, \dots, M$. In order to obtain a real CKM matrix in a natural way, it is particularly important to require CP invariance of the Lagrangian, only spontaneously broken by the vacuum. Without loss of generality, we may therefore assume that the above Yukawa

³ For a concise review of models with CP violation arising from the Higgs sector see, e.g., Ref. [14].

matrices are real, while the vacuum expectation values (VEVs) of the neutral Higgs fields are given by $\langle \phi_j^0 \rangle = v_j \exp(i\alpha_j)$, with $v^2 = \sum_j v_j^2 = (\sqrt{2}G_F)^{-1}$. Defining the Hermitian matrices

$$H_u \equiv M_u M_u^\dagger = \frac{1}{2} \left\{ \sum_i v_i^2 \tilde{\Gamma}_i \tilde{\Gamma}_i^T + \sum_{j>i} v_i v_j \left[\cos(\alpha_j - \alpha_i) (\tilde{\Gamma}_i \tilde{\Gamma}_j^T + \tilde{\Gamma}_j \tilde{\Gamma}_i^T) + i \sin(\alpha_j - \alpha_i) (\tilde{\Gamma}_i \tilde{\Gamma}_j^T - \tilde{\Gamma}_j \tilde{\Gamma}_i^T) \right] \right\} , \quad (24)$$

and

$$H_d \equiv M_d M_d^\dagger = \frac{1}{2} \left\{ \sum_i v_i^2 \Gamma_i \Gamma_i^T + \sum_{j>i} v_i v_j \left[\cos(\alpha_j - \alpha_i) (\Gamma_i \Gamma_j^T + \Gamma_j \Gamma_i^T) + i \sin(\alpha_j - \alpha_i) (\Gamma_j \Gamma_i^T - \Gamma_i \Gamma_j^T) \right] \right\} , \quad (25)$$

where M_u and M_d are the non-diagonal up- and down-type mass matrices respectively, one can show that within the three-generation SM a necessary and sufficient condition for CP invariance in the charged gauge interactions is given by [15]

$$T \equiv \text{Tr} [H_u, H_d]^3 = 0 . \quad (26)$$

Note that this is independent of the number of Higgs doublets which generate masses for the fermions. Hence, $T \neq 0$ implies CP violation mediated by charged gauge interactions, while $T = 0$ guarantees a real CKM matrix.

From Eq. (26) it is evident that the simplest and most natural way of obtaining a real CKM matrix (i.e. without any fine-tuning) is by introducing additional symmetries with the effect that

$$\Gamma_j \Gamma_i^T = \Gamma_i \Gamma_j^T \quad \text{and} \quad \tilde{\Gamma}_i \tilde{\Gamma}_j^T = \tilde{\Gamma}_j \tilde{\Gamma}_i^T . \quad (27)$$

Various multi-Higgs-doublet models where the condition in Eq. (27) is satisfied as a result of a discrete symmetry have been discussed in the literature. For example, the simplest models belonging to this class contain two Higgs doublets and a Z_2 symmetry [16] or three Higgs doublets and a Z_3 symmetry [17]. In these models, there are flavour-changing neutral currents and the vacuum leads to spontaneous CP violation, but the CKM matrix is real as a result of the discrete symmetry.

3.2 *Supersymmetric standard model*

The supersymmetric SM with spontaneous CP violation (SCPV) is another example where the CKM matrix is real provided that only two Higgs doublets are introduced. In the minimal supersymmetric standard model (MSSM), the tree-level vacua are all CP conserving so that SCPV can only occur if radiative corrections to the Higgs potential are taken into account. This inevitably leads to a mass of the lightest neutral Higgs boson which has already been ruled out by experiment [18]. On the other hand, in the next-to-minimal supersymmetric standard model (NMSSM) with one or more singlet fields in addition to the two doublets [19], SCPV can occur even at tree level. Whether or not this scenario is still consistent with present experimental bounds on the Higgs mass depends on the specific model, and we refer the interested reader to Ref. [20]. The important point to emphasize is that in the supersymmetric SM, provided there are only two Higgs doublets, but allowing for an arbitrary number of singlets, the CKM matrix is real, even if there is a physically meaningful relative phase between the VEVs of the two doublets. This can be readily verified by noting that the above phase can be eliminated from the quark mixing matrix [21].

4 A specific model

4.1 *Two-Higgs-doublet model*

For definiteness, we will consider a minimal extension of the SM with SCPV and a real CKM matrix as a result of a Z_2 symmetry [16]. Our main aim is to show that, within the framework of a specific two-Higgs-doublet model (2HDM), it is possible to have a real CKM matrix in a natural way, with ϵ_K and Δm_{B_d} generated by physics beyond the SM, while taking into account the constraints discussed in Sec. 2. As we shall see below, CP violation arises solely from the exchange of neutral Higgs bosons, thereby inducing flavour-changing neutral current (FCNC) interactions at tree level.

We start with the mass matrices M_u and M_d which can be diagonalized by means of a biunitary transformation, namely

$$p_L \rightarrow V_L^u u_L, \quad n_L \rightarrow V_L^d d_L, \quad p_R \rightarrow V_R^u u_R, \quad n_R \rightarrow V_R^d d_R, \quad (28)$$

so that

$$M_d^{\text{diag}} = V_L^{d\dagger} M_d V_R^d, \quad M_u^{\text{diag}} = V_L^{u\dagger} M_u V_R^u, \quad (29)$$

where

$$M_u = \frac{1}{\sqrt{2}}(v_1 \tilde{\Gamma}_1 + v_2 e^{-i\alpha} \tilde{\Gamma}_2), \quad M_d = \frac{1}{\sqrt{2}}(v_1 \Gamma_1 + v_2 e^{i\alpha} \Gamma_2). \quad (30)$$

In order to obtain the physical Higgs fields, it is useful to parametrize ϕ_1 and ϕ_2 in Eq. (23) as follows

$$\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\varphi_1^+ \\ v_1 + \rho_1 + i\eta_1 \end{pmatrix}, \quad \phi_2 = \frac{e^{i\alpha}}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\varphi_2^+ \\ v_2 + \rho_2 + i\eta_2 \end{pmatrix}. \quad (31)$$

The pseudo-Goldstone bosons G^+ and G^0 in our model can then be found by introducing new bases, i.e.

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = O \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ I \end{pmatrix} = O \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}, \quad \begin{pmatrix} H^0 \\ R \end{pmatrix} = O \begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix}, \quad (32)$$

with

$$O = \frac{1}{v} \begin{pmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{pmatrix}. \quad (33)$$

The mass matrix of the neutral Higgs fields is diagonalized through an orthogonal matrix U relating H^0 , R , and I to the mass eigenstates H_i ($i = 1, 2, 3$) via the relations

$$H^0 = \sum_{i=1}^3 U_{1i} H_i, \quad R = \sum_{i=1}^3 U_{2i} H_i, \quad I = \sum_{i=1}^3 U_{3i} H_i. \quad (34)$$

The Higgs-boson interactions with the quarks are then governed by the following Lagrangian

$$\begin{aligned} \mathcal{L}_Y = & (2\sqrt{2}G_F)^{1/2} \left\{ [\bar{u}(\Gamma^{u\dagger} V_{\text{CKM}} P_L - V_{\text{CKM}} \Gamma^d P_R) d] H^+ + \text{H.c.} \right\} \\ & + (\sqrt{2}G_F)^{1/2} \sum_{i=1}^3 \left\{ -U_{1i} H_i [\bar{d} M_d^{\text{diag}} d + \bar{u} M_u^{\text{diag}} u] \right. \\ & \quad - U_{2i} H_i [\bar{d}(\Gamma^d P_R + \Gamma^{d\dagger} P_L) d + \bar{u}(\Gamma^u P_R + \Gamma^{u\dagger} P_L) u] \\ & \quad \left. + i U_{3i} H_i [\bar{u}(\Gamma^u P_R - \Gamma^{u\dagger} P_L) u - \bar{d}(\Gamma^d P_R - \Gamma^{d\dagger} P_L) d] \right\}, \quad (35) \end{aligned}$$

where $V_{\text{CKM}} \equiv V_L^{u\dagger} V_L^d$ is the usual CKM matrix, $P_{L,R} = (1 \mp \gamma_5)/2$, and

$$\Gamma^u = V_L^{u\dagger} \left(\tilde{\Gamma}_1 \frac{v_2}{\sqrt{2}} - e^{-i\alpha} \tilde{\Gamma}_2 \frac{v_1}{\sqrt{2}} \right) V_R^u, \quad \Gamma^d = V_L^{d\dagger} \left(\Gamma_1 \frac{v_2}{\sqrt{2}} - e^{i\alpha} \Gamma_2 \frac{v_1}{\sqrt{2}} \right) V_R^d. \quad (36)$$

In order for the above Yukawa coupling matrices to fulfil the relation of Eq. (27), we impose an extra discrete Z_2 symmetry on the quark sector [see Eq. (23)] under which $\phi_2 \rightarrow -\phi_2$ and $n_{R3} \rightarrow -n_{R3}$, while all other fields remain unchanged.⁴ Denoting the ratio of the VEVs of the two Higgs bosons by $\tan \beta \equiv v_2/v_1$, we find

$$\Gamma^u = M_u^{\text{diag}} \tan \beta. \quad (37)$$

Hence there are no FCNC interactions in the up-quark sector (i.e. no CP violation in D^0 - \bar{D}^0 mixing).

Turning to the down-quark sector, the CKM matrix can be made real⁵ by redefining the phase of the quark field through $n_{R3} \rightarrow e^{-i\alpha} n_{R3}$ so that $V_L^{u,d}$ and $V_R^{u,d}$ are real and orthogonal matrices, while Γ_d takes the explicit form

$$\Gamma^d = \begin{pmatrix} m_d(\tan \beta - Sx^2) & -m_d S\beta_K & -m_d S\beta_d \\ -m_s S\beta_K & m_s(\tan \beta - Sy^2) & -m_s S\beta_s \\ -m_b S\beta_d & -m_b S\beta_s & m_b(\tan \beta - Sz^2) \end{pmatrix}. \quad (38)$$

Here $S = \tan \beta + \cot \beta$, with $S \geq 2$, and the β_n ($n = K, d, s$) are defined as

$$\beta_K = xy, \quad \beta_d = xz, \quad \beta_s = yz, \quad (39)$$

where x, y, z refer to the elements of the third column of $V_R^{d\dagger}$ with $x^2 + y^2 + z^2 = 1$. Consequently, the flavour-changing couplings of the three real neutral Higgs fields are constrained to obey

$$\beta_K \beta_d \beta_s = (\beta_K \beta_d)^2 + (\beta_K \beta_s)^2 + (\beta_d \beta_s)^2, \quad |\beta_n| \leq 1/2. \quad (40)$$

⁴ An alternative assignment is to choose both n_{R3} as well as p_{R3} to be odd under Z_2 . The advantage of this choice is that it leads to a naturally vanishing parameter $\bar{\theta} \equiv \theta_{\text{QFD}} + \theta_{\text{QCD}}$ at tree level, where $\theta_{\text{QFD}} = \arg[\det(M_u M_d)]$.

⁵ The one-loop corrections to the parameter J_{CP} [22] are equal to zero within the 2HDM in question [23].

The crucial point here is that the mass matrix M_d , given in Eq. (29), is in general not a Hermitian matrix and consequently $V_R^d \neq V_L^d$, which in turn implies that V_R^d is unrelated to V_{CKM} . As a result, the β_n above can be taken as free parameters, constrained only by condition (40). This is an important feature because the strength of the neutral Higgs contributions to Δm_K and $\Delta m_{B_{d,s}}$ (see below) are proportional to β_K and $\beta_{d,s}$ respectively. In fact, the neutral Higgs exchange induces a new physics contribution, in addition to the SM effective $\Delta S = 2$ Hamiltonian in Eq. (2), given by

$$H_{\text{eff}}^{\text{New}} = -\frac{G_F}{\sqrt{2}} m_s^2 (\beta_K S)^2 \sum_{i=1}^3 \frac{1}{M_i^2} \times \left\{ C_i^{LL} (\bar{d} P_L s)^2 + C_i^{RR} (\bar{d} P_R s)^2 + C_i^{LR} (\bar{d} P_L s) (\bar{d} P_R s) \right\} + \text{H.c.} , \quad (41)$$

with

$$C_i^{LL} = 2(U_{2i} - iU_{3i})^2, \quad C_i^{RR} = 2\zeta^2(U_{2i} + iU_{3i})^2, \quad C_i^{LR} = 4\zeta(U_{2i}^2 + U_{3i}^2), \quad (42)$$

where we have introduced the shorthand $\zeta = m_d/m_s$. The effective Hamiltonian inducing $B_q^0 - \bar{B}_q^0$ mixing can be obtained from the above by replacing

$$s \rightarrow b, \quad m_s \rightarrow m_b, \quad \beta_K \rightarrow \beta_d; \quad s \rightarrow b, \quad m_s \rightarrow m_b, \quad m_d \rightarrow m_s, \quad \beta_K \rightarrow \beta_s, \quad (43)$$

for the B_d^0 and B_s^0 system respectively.

To estimate the order of magnitude of the hadronic matrix elements resulting from Eq. (41), we rely on the vacuum insertion approximation. Defining the F -meson decay constant f_F ($F = K, B_q$) via

$$\langle F^0 | \bar{q} \gamma_\mu \gamma_5 q' | 0 \rangle \langle 0 | \bar{q} \gamma^\mu \gamma_5 q' | \bar{F}^0 \rangle = f_F^2 m_F^2, \quad q = d, s, \quad q' = b, s, \quad q' \neq q, \quad (44)$$

and employing the equation of motion, we get

$$\langle F^0 | \bar{q} \gamma_5 q' | 0 \rangle \langle 0 | \bar{q} \gamma_5 q' | \bar{F}^0 \rangle = - \left(\frac{m_F}{m_q + m_{q'}} \right)^2 f_F^2 m_F^2. \quad (45)$$

Thus, we obtain for the relevant matrix elements of the four-quark operators

$$\langle F^0 | (\bar{q} P_L q') (\bar{q} P_R q') | \bar{F}^0 \rangle = \frac{1}{2} \left[\left(\frac{m_F}{m_q + m_{q'}} \right)^2 + \frac{1}{6} \right] f_F^2 B_{1F} m_F^2, \quad (46)$$

$$\langle F^0 | (\bar{q} P_{L,R} q') (\bar{q} P_{L,R} q') | \bar{F}^0 \rangle = -\frac{5}{12} \left(\frac{m_F}{m_q + m_{q'}} \right)^2 f_F^2 B_{2F} m_F^2, \quad (47)$$

where B_{iF} are the bag parameters. In what follows we will use $B_{iF} = 1$, as determined by the vacuum insertion approximation, which is sufficient for our purposes.

The new physics contribution to M_{12} in the K system, Eq. (41), then takes the form

$$\begin{aligned} M_{12}^{\text{New}}|_K &= \frac{G_F}{\sqrt{2}} m_s^2 (\beta_K S)^2 f_K^2 m_K \sum_{i=1}^3 \frac{1}{M_i^2} \\ &\times \left\{ \left(\frac{5}{12} [(U_{2i} - iU_{3i})^2 + \zeta^2 (U_{2i} + iU_{3i})^2] - \zeta (U_{2i}^2 + U_{3i}^2) \right) \left(\frac{m_K}{m_s + m_d} \right)^2 \right. \\ &\left. - \frac{1}{6} \zeta (U_{2i}^2 + U_{3i}^2) \right\}, \end{aligned} \quad (48)$$

which in the limit $\zeta \ll 1$ reduces to

$$M_{12}^{\text{New}}|_K \simeq \frac{5}{12} \frac{G_F}{\sqrt{2}} f_K^2 m_K^3 (\beta_K S)^2 \sum_{i=1}^3 \frac{1}{M_i^2} (U_{2i}^2 - U_{3i}^2 - 2iU_{2i}U_{3i}). \quad (49)$$

As can be seen from Eqs. (43) and (49), the phase in the off-diagonal element M_{12} is common to both the K and B system, and we may therefore write

$$M_{12}^n = |M_{12}^{\text{New}}|_n e^{i\delta} + M_{12}^{\text{SM}}|_n \equiv M_{12}^{\text{SM}}|_n r_n^2 e^{2i\theta_n}, \quad n = K, d, s, \quad (50a)$$

with

$$\theta_n = \frac{1}{2} \arctan \left(\frac{R_n \sin \delta}{1 + R_n \cos \delta} \right), \quad r_n^2 = \sqrt{1 + 2R_n \cos \delta + R_n^2}, \quad (50b)$$

where $R_n \equiv |M_{12}^{\text{New}}|_n / M_{12}^{\text{SM}}|_n$.

4.2 Constraints from ϵ_K , Δm_K , and Δm_{B_d}

We now proceed to discuss various constraints on the parameters of the 2HDM. The observed CP violation in $K^0 - \bar{K}^0$ mixing is described by (for $\epsilon' \ll \epsilon_K$)

$$\epsilon_K \simeq \frac{e^{i\pi/4}}{\sqrt{2}} \left(\frac{\text{Im } M_{12}^K}{\Delta m_K} \right), \quad (51)$$

whereas the mass difference in the K system has the form

$$\Delta m_K \simeq 2|\text{Re } M_{12}^K| . \quad (52)$$

Using the experimental information on ϵ_K and Δm_K , Eqs. (16) and (17), as well as the central value $f_K = 160$ MeV, we obtain

$$(\beta_K S)^2 \sum_{i=1}^3 \frac{U_{2i} U_{3i}}{(M_i/\text{TeV})^2} \simeq 5.2 \times 10^{-4} , \quad (53)$$

and

$$(\beta_K S)^2 \sum_{i=1}^3 \frac{U_{2i}^2 - U_{3i}^2}{(M_i/\text{TeV})^2} \lesssim 0.16 . \quad (54)$$

A few remarks are in order here. First of all, it can be readily shown that [16]

$$\sum_{i=1}^3 \frac{U_{2i} U_{3i}}{M_i^2} \propto \cot \alpha . \quad (55)$$

Second, an analysis of the Higgs potential shows that in the limit of an exact Z_2 symmetry, the vacuum does not violate CP and the minimum of the potential is at $\alpha = \pi/2$, so that $\cot \alpha = 0$. Allowing only a soft breaking of the Z_2 symmetry by a dimension-two term in the Higgs potential, spontaneous CP violation can be generated [16,24]. This in turn implies that one may have $\cot \alpha \ll 1$ in a natural way (in the sense of 't Hooft [25]), since the limit $\cot \alpha = 0$ corresponds to the restoration of an exact Z_2 symmetry of the Lagrangian. This is the mechanism proposed by Branco and Rebelo [24] in order to naturally suppress the strength of CP violation. Therefore, the limit of Eq. (53), which is due to the ϵ_K measurement, does not necessarily imply very large values of the Higgs mass. On the other hand, we do not consider any accidental cancellation between the elements U_{2i} and U_{3i} appearing in Eq. (54) because it would require additional fine-tuning of the parameters in the Higgs potential. Furthermore, we should emphasize here that there are also contributions from box diagrams with W -boson and non-SM charged Higgs particle exchange, as well as long-distance contributions, which affect only the *real part* of M_{12}^K . Thus the most stringent limit on M_i comes from the experimentally measured value of Δm_K , resulting in the upper bound of Eq. (54). So, taking $(\beta_K S)$ to be of order unity, the Higgs masses have to satisfy $M_i \gtrsim \text{few TeV}$, whereas if we assume $(\beta_K S) \sim \mathcal{O}(10^{-1})$, then it is sufficient to demand that $M_i \gtrsim 250$ GeV.

Turning to the B system, the mass difference is given by

$$\Delta m_{B_q} = 2|M_{12}^q|, \quad q = d, s, \quad (56)$$

which is valid for $\Gamma_{12}^q/M_{12}^q \ll 1$. Recall from Sec. 2 that, in order for a model with a real CKM matrix to be consistent with present experimental data, a new physics contribution to Δm_{B_d} equalling at least 20% of the SM contribution is necessary when $\rho > 0$. It can be shown that such a new contribution due to $M_{12}^{\text{New}}|_d$ can be obtained in our model, while satisfying the bound of Eq. (54). In fact, using Eqs. (6), (43), (49), (50), and (56), as well as central values for the various input parameters, one finds that even $(\beta_d/\beta_K)^2 \sim \mathcal{O}(1)$ yields the necessary new physics contribution to Δm_{B_d} . Moreover, it is clear from the preceding discussion that there is a relation between θ_d and θ_K in the above 2HDM. Indeed, employing Eqs. (49)–(51), and (56), we obtain

$$\sin 2\theta_d = 2\sqrt{2}|\epsilon_K| \frac{\Delta m_K}{\Delta m_{B_d}} \left(\frac{m_{B_d}}{m_K}\right)^3 \left(\frac{f_{B_d}}{f_K}\right)^2 \left(\frac{\beta_d}{\beta_K}\right)^2, \quad (57)$$

and taking $f_K = 160$ GeV, $f_{B_d} = 200$ MeV, we estimate

$$\left(\frac{\beta_d}{\beta_K}\right)^2 \leq 7.4. \quad (58)$$

We see that, due to the enhancement factor of $(m_{B_d}/m_K)^3$ in Eq. (57), CP violation in B_d^0 -meson decays is not necessarily small. Before proceeding to discuss the CP asymmetries, we should mention that a more complete analysis of CP violation and physics beyond the SM has to take into account the recent measurement of ϵ'/ϵ_K [26]. However, as pointed out by Buchalla *et al.* [27] in the context of a specific 2HDM, a thorough investigation of the various contributions to direct CP violation in kaon decays, including renormalization group effects, is more involved and beyond the scope of the present paper.⁶

4.3 CP asymmetries and the 2HDM

We now turn to a study of the CP asymmetries. Due to the fact that the new physics contribution has a complex phase, the CP asymmetries in different B -meson decay channels are also affected. For example, using Eq. (57) and the experimental value of Eq. (16), we obtain for the B_d^0 system

⁶ See also the discussion in Ref. [16].

$$\sin 2\theta_d \simeq 0.13 \left(\frac{\beta_d}{\beta_K} \right)^2 \left(\frac{f_{B_d}}{200 \text{ MeV}} \right)^2 \left(\frac{160 \text{ MeV}}{f_K} \right)^2 . \quad (59)$$

Recall from Eq. (11) that in the case of a real CKM matrix, with $\alpha = \pi$ and $\beta = \gamma = 0$, the CP asymmetries take the form

$$a_{J/\psi K_S^0} = \sin 2(\theta_d + \theta_K), \quad a_{\pi^+\pi^-} = \sin 2(\pi - \theta_d) , \quad (60)$$

and assuming the limit $\theta_K \ll \theta_d$, one finds

$$a_{J/\psi K_S^0} = -a_{\pi^+\pi^-} . \quad (61)$$

We should emphasize that this result is a special feature of multi-Higgs-doublet models in which the CKM phase is absent, provided the new physics contributions cannot compete with the W -mediated tree diagrams of the SM. Note that due to the arbitrariness of the ratio β_d/β_K in Eq. (59), the CP asymmetries in this model can in principle vary from zero to one. This should be compared with the SM estimate which predicts $a_{J/\psi K_S^0}$ to lie within the range $0.5 \leq a_{J/\psi K_S^0} \leq 0.9$ [5,28]. Once the two asymmetries $a_{J/\psi K_S^0}$ and $a_{\pi^+\pi^-}$ are measured, Eq. (61) will provide a clear test of the class of models where CP violation arises exclusively from flavour-changing neutral Higgs exchange.

5 Discussion and conclusions

One of the fundamental open questions in particle physics concerns the origin of CP violation. In particular, it is crucial to verify whether charged weak interactions violate CP or not.

We have analysed the viability of a real CKM matrix, considering our present knowledge of the CKM matrix, derived from strange and B -meson decays, the experimental value of Δm_{B_d} , and the improved bound on Δm_{B_s} . We have shown that if one assumes that physics beyond the SM contributes only to ϵ_K , then the real CKM matrix is clearly disfavoured by the above-mentioned data. However, if one makes the more plausible assumption that new physics also contributes to Δm_{B_d} (and Δm_{B_s}), a real CKM matrix is still in keeping with present data. As a matter of fact, in order to fit the currently available data, it is crucial to have a new contribution to Δm_{B_d} corresponding to at least 20% (for $\rho > 0$), while a new contribution to Δm_{B_s} is not relevant at this stage. This is because only an experimental lower bound on the oscillation frequency in the B_s^0 system is available.

We have presented a general analysis which is quite model independent and ap-

plicable to a large class of models. This includes the supersymmetric SM with spontaneous CP violation, which has a real CKM matrix provided that only two standard Higgs doublets are introduced, apart from an arbitrary number of Higgs singlets. It is worth mentioning that within the supersymmetric SM it is possible to have the required additional contribution to Δm_{B_d} [29].

Moreover, we have shown that the assumption of spontaneous CP violation plays an extremely important role when the naturalness of a real CKM matrix is studied. For illustration, we have investigated a specific two-Higgs-doublet model where CP is spontaneously broken and the CKM matrix is real as a result of a Z_2 symmetry. In fact, we have shown that one can generate ϵ_K with simultaneous contributions to $\Delta m_{B_{d,s}}$ and obtain a good fit to the data.

We are eagerly awaiting the upcoming experiments at the various B factory facilities which will provide further constraints on the standard mechanism of CP violation as well as on alternative scenarios of CP breaking.

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